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EVALUATION OF THE SCATTER OF LIQUID LAUNCH VEHICLE POGO OSCILLATION AMPLITUDES DUE TO THE INFLUENCE OF THE SCATTER OF INTERNAL FACTORS

Almost all liquid launch vehicle developers faced the problem of ensuring stability in relation to POGO oscillations. The level of POGO amplitudes oscillations of the launch vehicle can be significantly affected by the scatter of internal factors. The study aims to create a mathematical model that can determine the range of POGO amplitudes in liquid launch vehicles. This will be demonstrated through the example of the Dnipro launch vehicle, which is affected by a variety of internal factors that cause its POGO amplitudes to vary. We developed the non-linear non-stationary mathematical model of POGO oscillations of the prototype of the Dnipro space launch vehicle. The model is built by taking into account the two lower vibration modes of the LV structure, two lower oscillation modes of the oxidizer feedline, and the first oscillation mode of the fuel feedline of the propulsion system. Modeling of dynamic processes was conducted in a combination of four liquid rocket engines based on the schematic of the staged rocket engine. The simulation takes into account cavitation phenomena in the engine pumps and delay times in the gas generators' chambers. We have developed a method for determining the scatter of the POGO oscillations caused by the action of internal factors, which is based on the use of the LP_τ *uniformly distributed sequences. As internal factors, the frequencies, decrements, and shapes of LV structural oscillation modes, the values of pressurization of the propellant tanks, and the engines' specific thrust impulses were considered. Based on the results of the calculations, the dependence of the POGO amplitudes in two regions of LV instability was determined, and the lower and upper enveloping curves for the POGO amplitudes were constructed. It is shown that the maximum POGO amplitudes oscillations in the first region of instability lie in the range from 0.23 g to 0.72 g and in the second region of instability — from 0 g to 0.60 g. Variants of combinations of internal factors, which provided the largest and smallest values of POGO amplitudes, were analyzed. This made it possible to determine the internal factors, the scatter of which has the greatest effect on the POGO amplitudes scatter: frequency, decrement, shape coefficients of oscillations of the oxidizer feedlines and the LV 1st mode structural longitudinal oscillations in the payload cross-section.*

Keywords: *liquid-propellant launch vehicle, POGO oscillations, liquid-propellant rocket propulsion system, mathematical modeling, internal factors, scatter of amplitudes of longitudinal vibrations.*

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INTRODUCTION

An integral component of the complex of problems solved in the case of creating liquid-propellant launch vehicles (LVs) is ensuring their longitudinal stability (POGO phenomenon). All developers of medium and heavy-class LVs have encountered this problem in the USA [23], in the USSR [7], in the European Union (France) [2], in Japan [27], in China [22], and in South Korea [4]. An increase in the amplitudes of longitudinal vibrations of the launch vehicle structure, which occurs during the loss of LV POGO stability, poses a threat to the integrity of structural elements and the execution of the LV flight program as a whole. In addition, as indicated in [11], despite the presence of general patterns inherent in the POGO phenomenon, each of the cases of loss of POGO stability of new launch vehicles has its own characteristics and is realized in a unique form. Currently, the creation of liquid launch vehicles is always accompanied by an analysis of their POGO stability and the development of measures to eliminate POGO vibrations (in case of LV instability) [3]. The most common means of eliminating the POGO instability of the launch vehicle or reducing the amplitudes of the POGO vibrations of the LV structure to acceptable values is the installation of POGO suppressors [19]. Such a planned and consistent solution to the problem of ensuring POGO stability was implemented during the creation of the Zenit liquid-propellant launch vehicle [15].

Currently, the development of methods for analyzing the POGO stability of liquid launch vehicles is directed towards a more detailed and complete account of physical processes and phenomena in the model of LV longitudinal vibrations. Thus, in [26], for analyzing the LV POGO stability, the propulsion systems of two propellants are studied. In [28], to take into account LV design features in more detail and simplify calculations, the main physical elements are divided and combined into nine types of generalized elements. In [29], to improve the results of POGO analysis, a method for three-dimensional modeling of a tank near the intake device was developed. In work [5], a finite element model of a coupled "propulsion – rocket structure" dynamic system was devised for POGO analysis. In [6], special attention is paid to taking into account cavitation phenomena in pumps based on the results of experimental studies.

Theoretical analysis of the POGO stability of liquid launch vehicles and the selection of parameters of POGO suppressors (if necessary) are carried out based on a study of the "LV structure – propulsion system" linear dynamic system. The corresponding linear mathematical model of POGO oscillations of the launch vehicle describes a wide range of physical processes and phenomena of various natures that occur during LV flight in the LV structure at different filling levels of its propellant tanks and in the propulsion system [24]. This mathematical model is very complex [19]. To determine the amplitudes of POGO oscillations of the launch vehicle, we need a nonlinear dynamic model of the "LV structure – propulsion system" that accounts for the effect of limiting the growth of the POGO vibration amplitudes due to the system's nonlinearity. The nonlinear model is more complex than the linear one and must also be nonstationary since the parameters of the longitudinal modes of the launch vehicle structure change significantly over the flight time [10, 11].

Due to the extreme complexity of developing a nonlinear model of POGO vibrations, the use of the model is observed in unique cases. These include the determination of the amplitudes of longitudinal vibrations of the Cyclone [10] and Dnepr [11] launch vehicles. Currently, the use of nonlinear models of longitudinal vibrations is justified in the case of modernizing existing launch vehicles that had acceptable values of the amplitudes of POGO vibrations.

Modernization of existing LVs for their commercial use requires significantly less time, material, and financial resources than the creation of new LVs. As a result of this modernization, the "Cyclone" and "Dnepr" liquid-propellant launch vehicles were developed by the Yuzhnoye Design Bureau and produced by the State Factory "Production Union Pivdennyi Machine-Building Plant named after O. M. Makarov" [1]. The prototype rockets of these launch vehicles were unstable to POGO vibrations, but the amplitudes of these vibrations had acceptable values [10, 11].

The modernization of these LVs is usually accompanied by an increase in the mass and dimensions of the payload and can lead to an increase in the amplitudes of POGO oscillations and an increase in their permissible level [12, 14]. However, before the launch of the modernized LVs, it is possible to estimate the amplitude of POGO oscillations only by computation [10, 11].

Note that the POGO amplitudes of the LV oscillations are significantly influenced by a number of internal factors, the scatter of which can significantly change the value of the parameters of the POGO vibrations of the launch vehicle. Such factors include the frequencies, shapes, and decrements of the natural longitudinal vibrations of the launch vehicle structure, the characteristics of the pressurization of its propellant tanks, and the characteristics of liquid rocket propulsion systems (LRPU). The variations in these factors are due to technological reasons, variations in the tank filling level, temperature and pressure of propellants, variations in the level of pressurization of propellant tanks and engine thrust.

The purpose of the article is to develop a mathematical model and determine the scatter of POGO amplitudes of liquid-propellant launch vehicles for the case of the Dnepr launch vehicle (as the model example), caused by the scatter of a set of internal factors.

1. MATHEMATICAL MODEL OF POGO OSCILLATIONS OF THE LAUNCH VEHICLE

In this work, the determination of the scatter in the amplitudes of POGO vibrations of liquid launch vehicles, caused by the scatter combination of internal factors, was carried out using the example of the prototype rocket of the Dnepr space launch vehicle [1]. During flight tests of this launch vehicle, it was found that in the active part of its flight during the propulsion system operation of the LV stage I there are two of the POGO instability time intervals (see Fig. 1). In the first time interval (from 1 s to 30 s of LV flight), the frequencies of POGO vibrations were in the range from 9 Hz to 12 Hz. In the second time interval (from 70 s to 100 s of flight), the oscillation frequencies were in the range of 15 Hz to 20 Hz. The maximum recorded values of the POGO structure amplitudes of the acceleration oscillations in the rocket's instrument compartment were 0.6 g in the first time interval and 0.20 g in the second time interval.

The layout diagram of the LV 1st stage propulsion system under consideration is shown in Fig. 2. Its

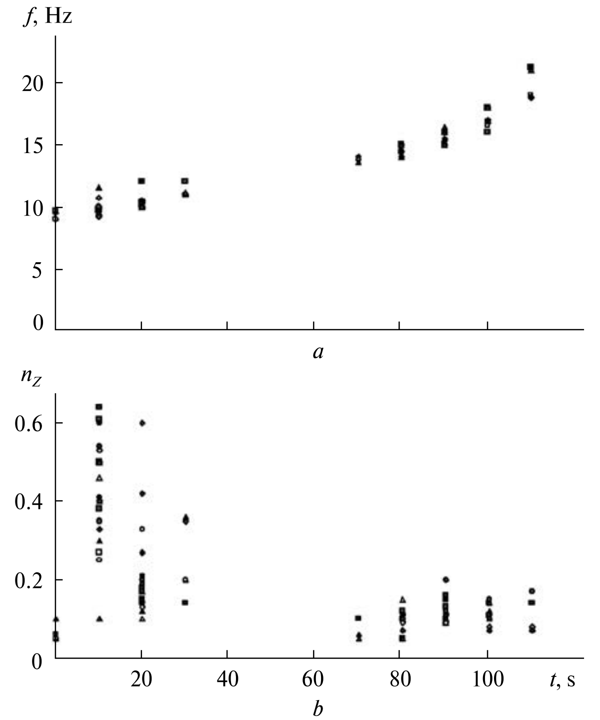

***Figure 1**.* Experimental frequencies (*a*) and amplitudes of the LV POGO acceleration oscillations (*b*) vs. the flight time of the first stage of the "Dnepr" launch vehicle [11]

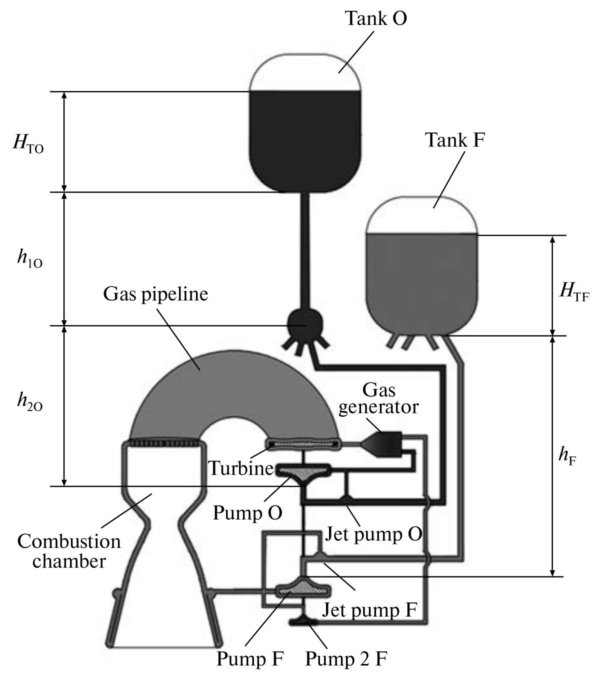

***Figure 2**.* The computation layout schematic of the first stage propulsion system of the launch vehicle

clustered propulsion system includes four main engines, designed according to the rich-oxidizer staged propulsion schematic. The engines are supplied with oxidizer from a common main pipeline. Fuel is supplied to the engines by autonomous pipelines.

Each engine includes an oxidizer pump (in Fig. 2 designated by pump O) and first- and second-stage fuel pumps (pump F, pump 2F), which are located on the same shaft of the turbine. The working gas for operating the turbine comes from the gas generator, where gasification of the entire oxidizer and combustion of the fuel, coming from the second stage fuel pump, occurs. After performing work on the turbine, the generator gas enters the combustion chamber through the gas duct, where it interacts with the main fuel flow and creates engine thrust. At the inlet to pumps O and F, the ejector pumps O and F (jet pump O, jet pump F) are installed to increase the cavitation margins of the main pumps O and F. The active fluid for the operation of the ejector pumps is taken from the outlet of the main pumps O and F. Presented in Fig. 2, filling levels H_{TO} and H_{TF} of the O and F tanks are variable during the LV flight. The projections of the feedlines O and F onto the vertical axis are indicated by h_{1O}, h_{2O}, and h_F.

In this work, the elastic longitudinal oscillations of the LV structure as a part of the "LV structure — propulsion system" dynamic system were described according to the basic principles outlined in the work [9]. The two lower modes of the longitudinal vibrations of the LV structure were taken into account, the frequencies of which are in the same frequency range as the frequencies of the two lower modes of the oxidizer feedline and the frequency of the first oscillation mode of the fuel feedline of LV propulsion system. The equations of the forced longitudinal oscillations of the LV structure relative to the flight path time for the *i*-th mode of structural oscillations have the form

$$\frac{d^2 z_i}{dt^2} + \frac{\omega_i(t)\delta_i(t,n_Z)}{\pi} \cdot \frac{dz_i}{dt} + \omega_i^2(t) z_i = \frac{\beta_e(t)\delta R_t}{m_{ri}(t)}, \quad (1)$$

where z_i is the longitudinal deviation of the *i*-th generalized coordinate of the LV structure relative to the position on the flight path, t is the LV flight time, $\omega_i(t)$ is the cyclic frequency of the *i*-th mode of the longitudinal oscillations of the LV structure, $\beta_e(t)$ is the shape of the *i*-th mode of the longitudinal oscillations of the LV structure at the place of attachment of the propulsion system, δR_t is the deviation of the total propulsion system thrust of the LV first stage, caused by the POGO oscillations of the LV, $m_{ri}(t)$ is the reduced mass of the mode of the longitudinal oscillations of the LV structure, $\delta_i(t,n_Z)$ is the dependence of the decrement of the *i*-th mode of the longitudinal oscillations of the LV structure on the flight time and the amplitude of the oscillations, which was set as follows:

$$\delta_i(t,\, n_Z) = \delta_i(t) + a \cdot n_Z^{\,2}, \quad (2)$$

$\delta_i(t)$ — the dependence of the decrement of the i-th mode of the longitudinal oscillations of the LV structure on the flight time, a is a constant coefficient determined from experimental data (a = 0.044).

The characteristics $\omega_i(t)$, $\delta_i(t)$, $\beta_e(t)$ and $m_{ri}(t)$ were determined for different values of the LV flight time according to work [9]. In this work, linear equations of longitudinal vibrations of the LV structure similar to (1) were used. The dependence of the decrement of longitudinal vibrations of the launch vehicle structure on the flight time and amplitude of vibrations $\delta_i(t,\, n_Z)$ are significant nonlinearities in the problem of longitudinal vibrations of liquid-propellant rockets [11]. Their use turns the equations of longitudinal vibrations of the launch vehicle structure into nonlinear ones.

It was believed that the deviation of the engine thrust δR_t is proportional to the deviation of the pressure in the engine combustion chamber

$$\delta R_t = R_t\,(p_c - \bar{p}_c)/p_c, \quad (3)$$

where p_c, $\bar{p}_c$ is the current and nominal pressure in the chambers of the cruising engine, R_t is the total propulsion system thrust of the LV first stage.

Modeling of fluid dynamics in feedlines, taking into account wave processes in the fluid, was carried out using the impedance method [16]. At the same time, the equivalence of models with distributed and concentrated parameters was ensured in the frequency range covering the frequencies of the two lower modes of the liquid natural oscillations.

The interaction of fluid oscillations in the engine feedlines and the vibrations of the rocket structure were taken into account in the equations of motion of the propellant in each section of the feedline:

$$p_{j+1} = p_j - \frac{a_j G_j^2}{\gamma_J} +$$
$$+ \gamma_j h_j \left(n_{cm}(t) + \frac{1}{g} \sum_{i=1}^{k} \beta_{ij}(t) \cdot \frac{d^2 z_i}{dt^2} \right) - J_j \frac{dG_j}{dt}, \quad (4)$$

where j is a pipeline section number, p_j, p_{j+1} are the pressure at the pipeline boundaries, a_j, J_j are coefficients of hydraulic and inertial pressure losses, G_j is the propellant flowrate at the j-th section of the pipeline, γ_j is the propellant specific weight, h_j is the height of the liquid column in the pipeline section; $n_{cm}(t)$ is the dependence of the overload of the LV center of mass on the flight time, which is determined by the results of ballistic calculations; g is the acceleration of gravity, $\beta_{ij}(t)$ is the dependence of the coefficient of the form of the j-th mode of the structural natural longitudinal vibrations on the time of flight for the j-th section of the pipeline, k is a number of considered modes of natural longitudinal oscillations of the LV structure.

Mathematical models of liquid rocket engines (LREs), used in the modeling of longitudinal oscillations of the launch vehicle, describe the dynamics of LREs at steady-state operating modes. The requirements for these models are significantly lower than for the models that describe the LRE dynamics in transient modes — in particular, at engine start-up and shutdown modes. However, a large number of various dynamic processes of different natures are observed even in steady-state engine operation mode. Mathematical modeling of these processes is the most difficult in the development of the “LV structure — propulsion system” dynamic system [3, 5]. The mathematical model of LRE low-frequency dynamics should describe liquid flows taking into account wave processes, subsonic and critical gas flows, thermophysical processes of gas formation, ignition and combustion in the gas generator and combustion chamber, mechanical processes during the operation of the turbopump unit, flow regulator and other units. Often, systems of equations describing low-frequency engine dynamics contain hundreds of differential and algebraic equations [18, 21].

A special place in the modeling of LRE dynamic processes is the taking into account cavitation phenomena in engine pumps. Due to their great flexibility, cavitation cavities with a relatively small volume completely change the LRE feedlines’ oscillation frequency. Taking into account cavitation phenomena in engine pumps has been consistent with the hydrodynamic model of cavitation pumps [16] with the results of the current experimental results from work [13]. In this case, the same form of equalizing the cavitation dynamics and equalizing the dynamic pressure at the outlet of the cavitation pump, like in the paper [17], was established:

$$(1+\alpha_p)\frac{dp_1}{dt} = \frac{G_1 - G_2}{C_b} + R_{b1}\frac{dG_1}{dt} + R_{b2}\frac{dG_2}{dt},$$
$$p_2 = p_1 + p_p \cdot \tilde{p}_p(V_b) - J_p \frac{dG_2}{dt}, \quad (5)$$

where p_1, G_1 are the pressure and fluid flowrate at the pump inlet, p_2, G_2 are the pressure and liquid flowrate at the pump outlet, $p_p, \tilde{p}_p(V_b)$ are the pressure and cavitation function of the pump, V_b is a cavitation volume, J_p is the coefficient of liquid inertial resistance in the flow part of the pump, α_p is a dynamic coefficient

$$\alpha_p = \partial(B_1 T_b) / \partial p_1 (G_1 - G_2);$$

$C_b = -\gamma / B_1$ is cavitation compliance, R_{b1}, R_{b2}, B_2 are cavitation resistance coefficients:

$$B_2(p_1, G_1) = \partial p_1 / \partial G_1,$$
$$R_{b1} = B_2 - B_1 \cdot T_b / \gamma + \partial p_{bd} / \partial G_1 - \partial(B_1 T_b) / \partial G_1 (G_1 - G_2),$$
$$R_{b2} = B_1 \cdot T_b / \gamma;$$

B_1, T_b — elasticity and the time constant of cavitation cavities, γ is the specific gravity of the liquid, p_{bd} is the pump breakdown pressure.

In LREs, based on a rich-oxidizer staged combustion schematic, the operating process in the gas generator must be described in terms of conversion time delay (of liquid propellant to combustion products) and the residence time of the gas in the ignited volume of a gas generator

$$\frac{dp_{gg}}{dt} = \frac{(RT)^*_{gg}}{V_{gg}} (G^*_{ggo} + G^*_{ggf} - G^*_t);$$
$$(RT)_{gg} = f(k_{gg}), \; k_{gg} = \frac{G^*_{ggo}}{G^*_{ggf}}, \quad (6)$$

$$\tau_c^{gg} \cdot \frac{dG^*_{ggo}}{dt} + G^*_{ggo} = G_{ggo}, \; \tau_c^{gg} \cdot \frac{dG^*_{ggf}}{dt} + G^*_{ggf} = G_{ggf}, \quad (7)$$

$$\tau_s^{gg} \cdot \frac{d(RT)_{gg}^*}{dt} + (RT)_{gg}^* = (RT)_{gg}, \tag{8}$$

$$G_t^* = F_{gg} \sqrt{g \frac{2\kappa_{gg}}{\kappa_{gg} - 1} \cdot \frac{p_{gg}^2}{(RT)_{gg}^*} \left[\left(\frac{p_{gc}}{p_{gg}} \right)^{\frac{2}{\kappa_{gg}}} - \left(\frac{p_{gc}}{p_{gg}} \right)^{\frac{\kappa_{gg}+1}{\kappa_{gg}}} \right]}, \tag{9}$$

where p_{gg}, $(RT)_{gg}$, κ_{gg} are the pressure, efficiency, and adiabatic index of combustion products in the gas generator, V_{gg}, F_{gg} are the volume and critical cross-sectional area of the gas generator, k_{gg} is the propellant ratio in the gas generator, G_{ggo}, G_{ggf} are flowrates of liquid propellant components in the LRE gas generator; G_t is the gas flowrate at the gas generator exit, p_{gc} is the gas pressure in the gas pipeline, τ_c^{gg} is a time of conversion of liquid propellant components into gaseous ones, τ_s^{gg} is the residence time of combustion gases in the generator.

Operation processes in the gas manifold are described by a system of equations similar to system (6)—(9).

For rich-oxidizer staged propulsion engines, taking into account the time delay of propellant gasification and the gas residence time in equations of the gas generator dynamics and the gas manifold is mandatory. It is important that, in such engines, the gas generator, gas duct, and turbopump unit create an additional closed internal engine combustion circuit. The time of mixing and the gas residence time τ_c^{gg}, τ_s^{gg} in the gas generator and gas manifold τ_c^{gc}, τ_s^{gc}, as well as the moment of turbopump unit rotor inertia, determine the frequency of the oscillating circuit. This circuit can strongly influence the engine gain factor, and errors in determining its characteristics can distort the results of determining the amplitudes of longitudinal oscillations. Delays in the equations of the dynamics of the combustion chamber do not have a noticeable effect on the engine dynamic gain since their values are much smaller, and the combustion chamber is not included in the specified circuit [18]. Work processes in the combustion chamber are described by a system of equations similar to (6)—(9), but without taking into account delays.

The complete system of equations (1)—(9), describing the POGO vibrations of the analyzed launch vehicle, contains more than 100 ordinary differential equations with variable coefficients, while the dynamic processes of each of the four rocket engines are modeled separately.

2. INTERNAL FACTORS INFLUENCING THE SCATTER OF AMPLITUDES OF POGO OSCILLATIONS OF THE LIQUID LAUNCH VEHICLE

Due to the uniqueness of the realization of the loss of POGO stability by different liquid LVs, the list of internal factors that affect the spread of amplitudes of POGO oscillations in each specific case may be different. In addition, the list of internal factors also depends on the calculation scheme and the content of the mathematical model of POGO oscillations of the launch vehicle.

In particular, in the “LV structure — propulsion system” dynamic system, the longitudinal oscillations of the LV structure can be modeled on the basis of various calculation schemes. If the modeling of the POGO oscillations of the rocket is performed taking into account the individual longitudinal modes of its structure (as in this case), then in the calculation scheme, the elastic oscillations of the structure are simulated by their mechanical analogues — several oscillators with damping — and are described accordingly by several equations of oscillating links. In this case, the internal factors influencing the amplitudes of POGO oscillations of the LV on the part of the structure are the frequencies, decrements, and shapes of its natural longitudinal oscillations. If a more detailed calculation scheme of the LV structure is used in the modeling of the POGO oscillations, in which it is represented by a branched chain or a complex multi-link system of finite elements (for example, [8]), then such factors are the elastic-mass characteristics and damping coefficients of the oscillations of these elements.

Internal factors should also include propellant tank pressures, which scatter of values affects cavitation phenomena in pumps and changes the natural frequencies of liquid oscillations in engine propulsion systems with oxidizer and fuel feedlines.

In [20], the influence of such internal factors as the temperature of the combustion products in the gas generator and the combustion chamber of the engines, the efficiency and pressure of the pumps, the area of the nozzle apparatus of the turbines, etc., are considered. It shows that the number of internal factors that significantly affect the thrust scatter of an engine installation can number several dozen, especially if the engine installation includes several engines. In this work, when determining the scatter of the POGO amplitudes of the LV, as internal factors related to the engines, we will consider only the specific thrust impulses J of the engines, which integrally contain the above-mentioned internal factors.

Let's assume that each of the engines, which is part of the clustered propulsion system, despite some geometric and operation mode differences of these engines, provides a given thrust P thanks to the LV control system. This can be achieved by changing the amount of propellant flow rates through the engines. From the formula for determining the specific impulse $J = P/G_\Sigma$ (where G_Σ is the total propellant flow rate through the engine), it is possible to determine the relationship between the scatter of the values of the specific impulse δJ and the scatter of the total propellant flowrate δG_Σ

$$\delta J = -(P / G_\Sigma^2)\delta G_\Sigma$$

or

$$\delta G_\Sigma = -(G_\Sigma^2 / P)\delta J \, .$$

In addition, we believe that during the propulsion system operation of the LV 1st stage, the flow rate ratio $k = G_O/G_F$ is ensured (where G_O, G_F are the flow rates of the oxidizer and fuel through the engine). Then the scatter of the oxidant and fuel flow rate values will be determined by the relations

$$\delta G_O = \delta G_\Sigma / (1 + 1/k)$$

and

$$\delta G_F = \delta G_\Sigma / (1 + k) \, .$$

Scatters of values of the specific impulse of the engine traction are determined by regulatory documents. Using them, we will get the propellant scatters of δG_O and δG_F, which can easily be used in the mathematical model of the "LV structure — propulsion system" dynamic system to determine the LV POGO amplitudes.

Table 1 presents the most significant internal factors, the effect of which was taken into account for determining the scatter of POGO amplitudes of the analyzed LVs, and their marginal deviations.

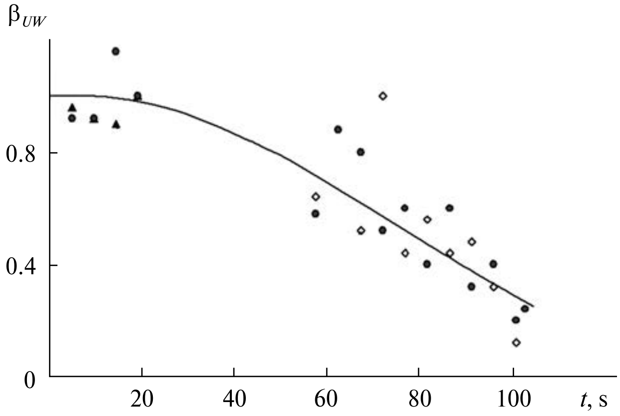

Figure 3. The first mode of longitudinal vibration shape of the launch vehicle structure at the payload section vs. the flight time of the launch vehicle first stage (experimental data and curve used in calculations)

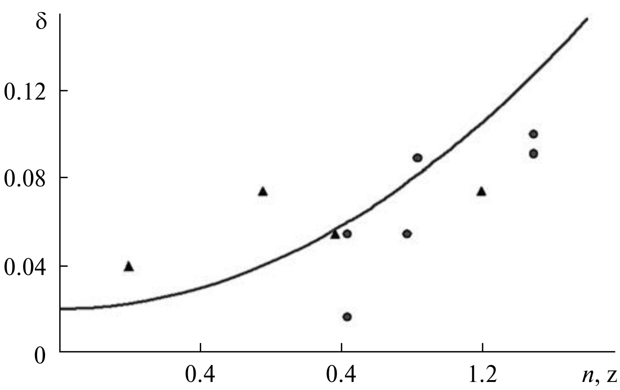

Figure 4. The first mode decrement of the of longitudinal vibrations of the launch vehicle structure vs. the longitudinal vibrations amplitude (experimental data and curve used in calculations)

Determining the scatter of internal factors is a separate task in general. In this work, the assessment of their limit values was carried out as follows. The scatter of natural frequencies of the longitudinal vibrations of the LV structure was estimated according to experimental data (see Fig. 1, *a*) and was 0.5 Hz. Fig. 3 presents the experimental values and the calculation curve of the dependence of the oscillation shape of the I mode of the longitudinal oscillations of the LV structure in the cross-section corresponding to the location of the LV payload, on the time of the rocket flight. The maximum scatter of its values, as well as the scatter of the values of other shapes of the

Table 1. **Internal factors taken into account for determining the scatter of POGO amplitudes of the LV**

No	Name of the factor	Limit deviations	Marking
1	The natural frequency of the 1st mode of the longitudinal oscillations of the LV structure, Hz	±0.5	x_1
2	The natural shape of the 1st mode of the longitudinal oscillations of the LV structure, which corresponds to the oxidizer feedline, %	±10	x_2
3	The natural shape of 1st mode of the longitudinal oscillations of the LV structure, which corresponds to the oxidizer tank, %	±10	x_3
4	The natural shape of 1st mode of the longitudinal oscillations of the LV structure, which corresponds to the fuel tank, %	±10	x_4
5	The natural shape of 1st mode of the longitudinal oscillations of the LV structure, which corresponds to the LV payload, %	±10	x_5
6	The decrement of the first mode of the natural longitudinal oscillations of the LV structure, %	±40	x_6
7	Pressure in the oxidizer tank, bar	±0.25	x_7
8	Pressure in the fuel tank, bar	±0.20	x_8
9	The oxidizer flow rate through the engine № 1, %	±4.0	x_9
10	Fuel flow rate through the engine № 1, %	±4.0	x_{10}
11	The oxidizer flow rate through the engine № 2, %	±4.0	x_{11}
12	Fuel flow rate through the engine № 2, %	±4.0	x_{12}
13	The oxidizer flow rate through the engine № 3, %	±4.0	x_{13}
14	Fuel flow rate through the engine № 3, %	±4.0	x_{14}
15	The oxidizer flow rate through the engine № 4, %	±4.0	x_{15}
16	Fuel flow rate through the engine № 4, %	±4.0	x_{16}

form of the I mode of the longitudinal vibrations of the LV structure (see Table 1), was taken to be equal to 10 % of the calculated nominal values. Experimental data and calculation curve (see Fig. 4) regarding the dependence of the decrement on the POGO amplitude of oscillations were used to estimate the scatter of the values of the decrement of the I mode of the longitudinal oscillations of the LV structure. According to this estimate, the maximum scatter of the values of the decrement of oscillations is 40 %. Scatters of pressures in propellant tanks (as well as scatters of the specific thrust impulse or propellant flow rates) were determined by regulatory documents.

The calculations showed that for the considered internal factors, the frequency and decrement of the longitudinal oscillations of the LV structure, as well as the shapes of natural longitudinal oscillations corresponding to the location of the oxidizer feed feedline, have the greatest influence (within the specified scatter) on the scatter of the POGO amplitudes. The pressures in the tanks of the propellants and the specific thrust impulses of the engines have the least influence. The degree of influence of the specified internal factors was assessed based on the results of calculations of the POGO amplitudes at extreme changes in the values of each individual parameter, according to Table 1.

3. THE METHOD OF DETERMINING THE SCATTER OF INTERNAL FACTORS

Let's assume that the nonlinear mathematical model of the "LV structure — propulsion system" dynamic system, based on which the POGO amplitudes of the liquid LV are evaluated, contains n internal factors $\alpha_1,\ldots,\alpha_n$. The set of points A_i with Cartesian coordinates $\alpha_{1,i},\ldots,\alpha_{n,i}$ forms an n-dimensional space of parameters. In fact, the values of the parameters $\alpha_1,\ldots,\alpha_n$ change within limited limits and belong to the admissible set $\mathbf{P}$, which is an n-dimensional region of the parameter space:

$$\mathbf{P} = \{ \alpha_1,\ldots,\alpha_n : \alpha_j^{\min} \le \alpha_j \le \alpha_j^{\max},\ j=1,\ldots,n \}. \quad (10)$$

A direct approach to determining the scatter of POGO amplitudes of liquid launch vehicles consists of implementing various combinations of values of external and internal factors from the admissible set (10). Values of imprecisely specified parameters (trial points) can be chosen based on a random search, while the search will be more effective the more uniformly their values are located in the parameter space [25]. In both cases, the solution to the problem of determining the scatter of POGO amplitudes of the liquid launch vehicle, due to the scatter of internal factors, requires a large number of calculations and is quite time-consuming, even with a relatively small number of varied parameters.

In this work, it is proposed to use a space probing method, which is called LP-search, to select test points $\alpha_1,\ldots,\alpha_n \in \mathbf{P}$. This method is implemented for a one-dimensional cube. The points of LPτ-sequences are used as test points in the n-dimensional cube. LPτ-sequences are the most evenly distributed among all currently known sequences. Their use has an advantage compared to the simplest random search and is much more efficient than uniform cubic lattices for $n > 1$ [25].

Different combinations of deviations of each of the internal factors determine various combinations of the values of the varied parameters when calculating the POGO amplitudes. Combinations of deviations of parameters reduced to a dimensionless form were determined by LPτ-sequences, which were constructed in a unit cube K^n according to [25]. The use of LPτ-sequences makes it possible to cover the deviations of internal factors more evenly with a smaller number of implementations. The calculation algorithm is presented in [25], and Table 2 shows a set of combinations of internal factors when calculating amplitudes of longitudinal oscillations obtained using LPτ-sequences. Each row of the table determines the coordinates $(x_1, x_2, \ldots, x_n)$ of one point of the n-dimensional unit cube and specifies one of the options for implementing the deviations of the internal factors. Each column of the table specifies one coordinate of a point in the n-dimensional unit cube for all variants of realizations of deviations of internal factors.

The data in Table 2 are used as follows. Let us denote the internal factors of the system given in Table 1, variables $y_1, y_2, \ldots, y_{16}$. Then, if the scatter of internal factors is specified in absolute values, the current value of the parameter y_i, chosen taking into account the possible scatter of its values, is determined by the formula

$$y_i = \overline{y}_i + 2\Delta x_i (x_i - 0.5),$$

where $\overline{y}_i$ is the nominal value of the parameter, Δx_i is the boundary deviation of the parameter from the nominal value, x_i is the number from the x_i-th column of Table 2.

If the scatter of internal factors is specified as a percentage of the nominal value, then the current value of the parameter is determined by the formula

$$y_i = \overline{y}_i\,[1 + 2\Delta x_i (x_i - 0.5)].$$

Table 2. **A set of scatters for combinations of internal factors for determining the scatter of POGO amplitudes of the liquid LV**

Variant number	Coordinates of points of LPτ-sequence							
	x_1	x_2	x_3	x_4	x_5	x_6	...	x_{16}
1	0.5	0.5	0.5	0.5	0.5	0.5	...	0.5
2	0.25	0.75	0.25	0.75	0.25	0.75	...	0.25
3	0.75	0.25	0.75	0.25	0.75	0.25	...	0.75
4	0.125	0.625	0.875	0.875	0.625	0.125	...	0.375
5	0.625	0.125	0.375	0.375	0.125	0.625	...	0.875
6	0.375	0.375	0.625	0.125	0.875	0.875	...	0.125
...	...	...	...	...	...	...	...	0.563
255	0.996	0.004	0.770	0.582	0.574	0.746	...	0.063
256	0.002	0.502	0.908	0.857	0.346	0.627	...	0.688

4. RESULTS OF DETERMINING THE SCATTER OF POGO AMPLITUDES

Based on the results of mathematical modeling of the developed "LV structure — propulsion system" non-linear non-stationary dynamic system, the dependence of the POGO amplitudes of the prototype of the "Dnipro" space LV on the flight time (during the operation of its engines of the LV 1st stage) with various combinations of scatters of internal factors was determined. According to the results of calculations, as well as according to the results of experiments, two-time intervals of instability of the system were obtained (see Fig. 5). Note that the LV instability of the investigated system in the first interval was obtained for all considered combinations of internal factor scatters, and the calculated limits of the first interval of LV instability are close to the experimental ones obtained from the results of the 20 rocket launches. The second interval of dynamic system instability is absent in some variants of the combination of scatters of internal factors. Compared with the experimental data, the calculated boundaries of the second interval are noticeably wider, and the calculated POGO amplitudes of the LV reach their maximum values somewhat earlier.

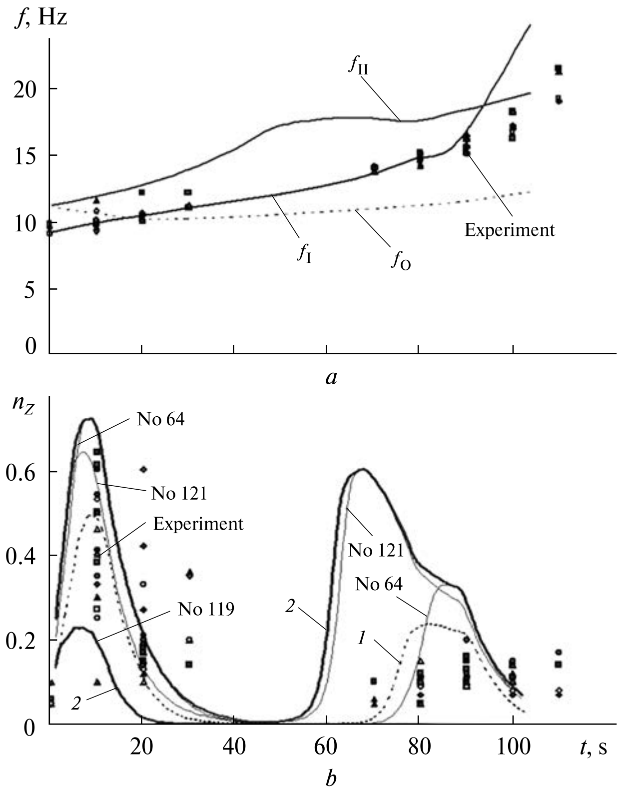

Figure 5. Oxidizer feedline oscillation frequencies f_O, the launch vehicle structure f_I, f_{II} (*a*) and longitudinal acceleration amplitudes (*b*) vs. the flight time of the first stage launch vehicle: *1* is without taking into account the internal factors spread, *2* is lower and upper envelope curves

The lower and upper enveloping curves were constructed for the dependence of the POGO amplitudes of the LV analyzed (see Fig. 5, *b*, curves *2*). It is shown that in the first time interval of POGO instability, the maximum values of calculated POGO amplitudes are in the range from 0.23 g to 0.72 g (the value 0.49 g is without taking into account the scatter of internal factors, curve *1* in Fig. 5, *b*), and in the second interval of POGO instability is in the range from 0 g to 0.60 g (the value 0.24 g is without taking into account the scatter of internal factors). These enveloping curves for the dependences of the POGO amplitudes in both flight time intervals of POGO instability cover most of the results of 20 launches of this liquid rocket (see Fig. 5, *b*, curves *2*).

Variants of combinations of scatters of internal factors were analyzed, in which the largest and smallest values of POGO amplitudes were obtained. In particular, it was established that in the first time interval of instability, the amplitudes reached their maximum values (option No. 64) when the frequency and decrement of the first mode of the POGO vibrations of the LV structure were close to the lower limit of the range of their possible values, and the shape coefficient of the structural vibrations in the section corresponding to the location of the instrument compartment is to the upper limit (see Table 3). The minimum values of POGO amplitudes (option No. 119) were recorded

Table 3. **Variants of combinations of the most important internal factors, in which the maximum and minimum values of the POGO amplitudes of the liquid LV are recorded**

Variant number	Coordinates of points of LPτ-sequence					
	x_1	x_2	x_3	x_4	x_5	x_6
64	0.008	0.664	0.523	0.383	0.977	0.195
119	0.930	0.055	0.227	0.336	0.180	0.961
121	0.617	0.617	0.164	0.148	0.992	0.023

when the frequency and decrement of I mode of the POGO oscillations of the LV structure were close to the upper limit of the range of their possible values, and the coefficients of the form of oscillations of the LV structure, which correspond to the location of the LV instrument compartment and the oxidizer feedline is to the lower limit (see Table 3). In the second time interval of instability, the maximum value of the amplitude (option No. 121) was reached for small (close to the lower limit) values of the decrement of the I mode of the LV structure longitudinal vibrations, as well as large (close to the upper limit) values of the oscillation shape, which corresponds to the location of the LV instrument compartment (see Table 3). Table 3 presents the value of only the first six internal factors, the influence of which is the greatest.

The influence of the scatter of the engine-specific thrust impulse does not have a significant effect on the scatter of the POGO amplitudes of the considered LV due to the relatively small value of this scatter. For the same reason, no noticeable differences were observed in the oscillations of engine parameters among themselves and the interaction of the oscillation systems of different engines.

CONCLUSIONS

1. A non-linear, non-stationary mathematical model of LV longitudinal oscillations for the prototype of the Dnipro space launch vehicle was developed. It took into account two lower modes of oscillations of the LV structure, two lower oscillation modes of the oxidizer feedline, and the first oscillation mode of the fuel feedline. Modeling of dynamic processes of the propulsion system with the four rich-oxidizer staged propulsion LRE was carried out, taking into account pump cavitation phenomena, gasification time delays of the propellants, and the gas residence time delays in gas generators.

2. A method for determining the scatter in the amplitudes of POGO vibrations of liquid launch vehicles due to the action of scatter combination of internal factors has been developed. Frequencies, decrements, and the shapes of the longitudinal natural vibrations of the launch vehicle structure, the propellant tanks' pressurization, and the specific thrust impulses of the engines were considered as internal factors. The method is based on the use of LPτ uniformly distributed sequences that is much more efficient compared to random search.

3. Based on the results of calculations, the dependence of the amplitudes of POGO oscillations of the considered launch vehicle in two LV flight time instability intervals was determined, and the lower and upper enveloping curves for the POGO amplitudes were constructed. It is shown that the maximum POGO amplitudes in the first time interval of LV instability lie in the range from 0.23 g to 0.72 g and in the second time of LV instability — from 0 g to 0.60 g. Variants of combinations of internal factor scatters, which obtained the largest and smallest amplitudes of continuous oscillations, were analyzed. This made it possible to determine the internal factors, the scatter of which has the greatest influence on the scatter of the POGO amplitudes of the LV: frequency, decrement, the shape of oscillations of the oxidizer feed feedline and in the section of the payload on the 1st mode of the longitudinal oscillations of the LV structure.

4. Taking into account the noticeable scatter in the amplitudes of the POGO vibrations of the Dnepr launch vehicle due to the scatter of internal factors, it is recommended that in case of determining the POGO amplitudes of other launch vehicles, it is recommended to carry out mathematical modeling not only at nominal operating modes but also to take into account the inevitable scatter of initial data, external conditions and characteristics of the research objects.

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ВИЗНАЧЕННЯ РОЗКИДУ АМПЛІТУД ПОЗДОВЖНІХ КОЛИВАНЬ РІДИННИХ РАКЕТ-НОСІЇВ ЗА РАХУНОК ВПЛИВУ РОЗКИДУ ВНУТРІШНІХ ФАКТОРІВ

З проблемою забезпечення стійкості щодо поздовжніх коливань рідинних ракет-носіїв (РН) стикалися практично всі розробники РН. На рівень амплітуд поздовжніх коливань (АПК) РН можуть суттєво впливати розкиди внутрішніх факторів. Метою статті є розробка математичної моделі та визначення розкиду амплітуд поздовжніх коливань рідинних ракет-носіїв на прикладі РН «Дніпро», викликаного розкидом сукупності внутрішніх факторів. У роботі розроблено нелінійну нестаціонарну математичну модель поздовжніх коливань прототипу космічної РН «Дніпро». У цій моделі враховувалися два нижчі тони коливань конструкції РН і два нижчі тони коливань окислювача і перший тон коливань пального. Моделювання динамічних процесів у зв'язці із чотирьох рідинних ракетних двигунів, виконаних за схемою з допалюванням генераторного газу, проведено з урахуванням кавітаційних явищ у насосах двигунів та часів запізнювань у вогневих порожнинах газогенераторів. Розроблено метод визначення розкиду АПК рідинних РН, зумовленого дією внутрішніх факторів, який базується на використанні рівномірно розподілених LPτ-послідовностей. В якості внутрішніх факторів були розглянуті частоти, декременти та коефіцієнти форм власних поздовжніх коливань корпуса РН, тиски наддуву паливних баків та питомі імпульси тяги двигунів. За результатами розрахунків визначено залежність АПК розглянутої РН у двох областях нестійкості та побудовано нижню і верхню обвідну криві для АПК. Показано, що максимальні амплітуди поздовжніх коливань у першій області нестійкості лежать в діапазоні від 0.23 до 0.72*g* і в другій області нестійкості — від 0 до 0.60*g*. Проаналізовано варіанти поєднань розкидів внутрішніх факторів, за яких було отримано найбільші та найменші значення АПК. Це дозволило визначити внутрішні фактори, розкид яких найбільше впливає на розкид АПК: частота, декремент, коефіцієнти форми коливань живильного трубопроводу окислювача і у перерізі корисного навантаження на I тоні поздовжніх коливань корпуса РН.

Ключові слова: рідинна ракета-носій, поздовжні коливання, рідинна ракетна двигунна установка, математичне моделювання, внутрішні фактори, розкид амплітуд поздовжніх коливань.