THE TRANSFORMATION OF LONG SCALE ALFVÉN WAVES IN SPACE DUSTY PLASMA

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A nonlinear mechanism of the generation of kinetic Alfvén waves (KAW) on dust plasma with small plasma parameter β is proposed. As the generation mechanism, the parametric instability where a pumping wave is the MHD Alfvén wave is considered. On the basis of the three-fluid MHD, the nonlinear dispersion equation describing the three-wave interaction is deduced and its solution is derived. Obtained instability growth rate is determined by parameters of dust plasma particles. The nonlinear process under consideration can take place both in laboratory and in space plasma with small plasma parameter β. As an application of theoretical results, we consider Saturn's F-ring.

INTRODUCTION

Now dust plasma is the object of the intensive researches from a set of application, both in laboratory experiments, and in space. The extensive scientific literature, including reviews [3, 4] and monographies [1, 6], shows large interest to dust plasma. Practically, any plasma (including space) contains some number of dust grains. The dust grains are ices, graphite, silicates, *etc.* The interplanetary space, rings of giant planets, tail of comets and, certainly, the Earth's magnetosphere and ionosphere are typical objects of Solar System which contain a lot of dust. Typical parameters of dust plasma in this regions are reduced in Table 1.

Table 1. Dust plasma parameters

Characteristics	Interplanetary space	The giant planets rings	Comet tail	Earth's ionosphere
n_e (cm^{-3})	≈ 5	$0.1 - 10^2$	$10^3 - 10^4$	$\approx 10^3$
T_e (K)	$\approx 10^5$	$10^5 - 10^6$	≤ 0.1 (eV)	≈ 150
n_d (cm^{-3})	$\approx 10^{-12}$	$10^{-7} - 10$	$10^{-10} - 10^{-3}$	$\approx 10 - 10^3$

The most important source of dust in the Earth's atmosphere is simulated contamination (aerosols). The presence of dust grains in the rings of the giant planets was established during satellite missions of Voyagers 1 and 2 [7, 8].

In the present work we consider nonlinear parametric interaction MHD Alfvén pump wave (ω_0, $\vec{k}_0$) with kinetic Alfvén waves ($\omega_1, \vec{k}_1$, ω_2, $\vec{k}_2$) in dust plasma. The frequency ω_0 and wave vector $\vec{k}_0$ are related by linear dispersion relation:

$$\omega_0^2 = k_{0z}^2 V_{Ad}^2,$$

where $V_{Ad} = B_0/\sqrt{4\pi n_d m_d}$ is the dust Alfvén velocity, n_d and m_d is the density and mass of dust grains. We consider equation in Cartezian coordinates (x, y, z), supposing that all wave vectors are situated in xz plane $\vec{k} = (k_x,\, 0,\, k_z)$, $\vec{B}_0 = B_0 \vec{e}_z$. It is assumed that the wave synchronism conditions are satisfied:

$$\omega_0 = \omega_1 + \omega_2,$$
$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2.$$

DISPERSION EQUATION FOR DUST KAW

Nonlinear three-wave interaction in the dust plasma is considered on the basis of magnetic hydrodynamics. In this case the main system of equations describes electrons, ions and charged dust particles as conductive liquids connected with each other by electromagnetic fields. Thus, the main system of equations looks like:

$$\frac{\partial \overrightarrow{V_\alpha}}{\partial t} = \frac{1}{m_\alpha}\left(e_\alpha \vec{E} + \vec{F_\alpha}\right) + \left(\overrightarrow{V}_\alpha \times \omega_{B\alpha}\right) - \frac{T_\alpha}{m_\alpha n_\alpha}\vec{\nabla} n_\alpha; \tag{1}$$

$$\frac{\partial n_\alpha}{\partial t} = -\vec{\nabla}\left(n_\alpha \overrightarrow{V_\alpha}\right); \tag{2}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}; \tag{3}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}; \tag{4}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho; \tag{5}$$

where $\vec{j} = e\left(n_i\vec{V_i} - n_e\vec{V_e} - Zn_d\vec{V_d}\right)$, $\rho = e\left(n_i - n_e - Zn_d\right)$, $\vec{F_\alpha} = \frac{e_\alpha}{c}\left(\vec{V_\alpha} \times \vec{B}\right) - m_\alpha\left(\vec{V_\alpha}\nabla\right)\vec{V_\alpha}$. Index $\alpha = i,\, e,\, d$ correspond to the ion, electron and dust (negative charge) plasma components. As the frequence range $\omega \ll \omega_{Bd},\, \omega_{Bi}$ is considered, the influence of displacement current is not essential.

Nonlinear dispersion equation for KAWs in electron-ion plasma was considered in [9, 10]. It is obvious that in case of three-component plasma this equation will be a little modified, as the plasma approximation in this case is of the form:

$$\tilde{n}_i = \tilde{n}_e + Z\tilde{n}_d, \tag{6}$$

where $\tilde{n}_e$, $\tilde{n}_i$, $\tilde{n}_d$ are perturbations of the ion, electron and dust grains number densities, respectively. This can be used, since the wave modes are considered at the frequencies less than ion cyclotron frequency. Using the motion (1) and continuity (2) equations we find:

$$\tilde{n}_e = \frac{en_{0e}}{k_z T_e}\left(k_z\varphi - \frac{\omega A_z}{c} + i\frac{F_z}{ek_z}\right); \tag{7}$$

$$\tilde{n}_i = -\frac{en_{0i}}{k_z T_i}\left(k_z\varphi - \frac{\omega A_z}{c}\right); \tag{8}$$

$$\tilde{n}_d = -\frac{Zen_{0d}}{m_d\omega^2}\left[\left(k_z^2 - \frac{\omega^2 k_x^2}{\omega^2 - \omega_{Bd}^2}\right)\varphi - \frac{k_z\omega}{c}A_z\right], \tag{9}$$

where φ and A_z are scalar potential and z-component of the vector potential KAW, $\omega_{Bd} = eZB_0/m_d c$ is the cyclotron frequency of grains with charge Ze.

Substituting (7)–(9) in (6) we obtain nonlinear dispersion equation:

$$\left[\left(\omega^2 - k_z^2 C_d^2\right)\left(\omega^2 - k_z^2 V_{Ad}^2\right) - k_z^2 V_{Ad}^2\,\mu_d\,\omega^2\right]\varphi = k_z^2 V_{Ad}^2\,\omega^2 Q_{NL}, \tag{10}$$

where

$$Q_{NL} = -i\frac{m_d n_{oe} C_d^2 F_z}{en_{0d}Z^2 k_z T_e}, \quad C_d = \left[\frac{Z^2 T_e T_i n_{0d}}{\left(n_{oe}T_i + n_{oi}T_e\right)m_d}\right]^{1/2}, \quad V_{Ad} = \frac{B_0}{\sqrt{4\pi n_{od} m_d}},$$

$\mu_d = k_\perp^2\rho_d^2$, $\rho_d = \dfrac{C_d}{\omega_{Bd}}$ is dust particle Larmor radius.

In the absence of pump wave ($Q_{NL} = 0$) and $\mu_d \neq 0$ we have:

$$\omega^2 = k_z^2 V_{Ad}^2\left(1 + \mu_d\right), \tag{11}$$

$$\omega^2 = \frac{k_z^2 C_d^2}{1 + \mu_d}. \tag{12}$$

The expression (11) describes the dispersion law for KAW, and (12) describes the dispersion law for ion-acoustic waves in dust plasma.

Taking into account that $V_{Ad} \gg C_d$ we can note the equation (10) as:

$$\varepsilon_A(\omega, k)\,\varphi = P_{NL}, \tag{13}$$

where $\varepsilon_A(\omega, k) = \omega^2 - k_z^2 V_{Ad}^2 (1 + \mu_d)$, $P_{Nl} = k_z^2 V_{Ad}^2 Q_{Nl}$.

From (13) we obtain KAW dispersion equation:

$$\varepsilon_1 \varphi_1 = \eta_1 \left(E_{0x} \varphi_2^*\right), \tag{14}$$

where coupling coefficient is:

$$\eta_1 = i \frac{e m_d n_{0e} C_d^2 V_{Ad}^2 \omega_2}{n_{0d} Z^2 T_e^2 \omega_0} \frac{k_{0z} k_{1z}}{k_{2x}} \mu_e.$$

The dispersion equation for second KAW follows from (14), where the index "1" and "2" are exchanged:

$$\varepsilon_2 \varphi_2 = \eta_2 \left(E_{0x} \varphi_1^*\right), \tag{15}$$

where

$$\eta_2 = i \frac{e m_d n_{0e} C_d^2 V_{Ad}^2 \omega_1}{n_{0d} Z^2 T_e^2 \omega_0} \frac{k_{0z} k_{2z}}{k_{1x}} \mu_e.$$

NONLINEAR DISPERSION EQUATION

Using dispersion equation for two KAW (14) and (15), we can find a nonlinear dispersion equation describing three-wave interaction:

$$\varepsilon_1 \varepsilon_2^* = \eta_1 \eta_2^* |E_{0x}|^2. \tag{16}$$

Assuming in (16) $\omega_1 = \omega_{1r} + i\gamma$, $\omega_2 = \omega_{2r} + i\gamma$ ($|\gamma| \ll \omega_{1r}$, ω_{2r}) and expanding ε_1 and ε_2 in Taylor series in the small parameter γ, we find:

$$\gamma = \frac{W}{2} \frac{(\pi n_{0i} T_i)^{\frac{1}{2}}\, e m_d n_{0e} C_d^2 V_{Ad}^2 k_{0z}}{n_{0d} Z_d^2 T_e^2 \omega_0} \left(\frac{k_{1z} k_{2z}}{k_{1x} k_{2x}}\right)^{\frac{1}{2}} \mu_e, \tag{17}$$

where $W = \dfrac{|E_{0x}|}{\sqrt{4\pi n_0 T_i}}$.

CONCLUSION

The nonlinear parametric interaction of the pump MHD Alfvén wave with kinetic Alfvén waves in dust plasma with small plasma parameter β is considered. To describe the nonlinear interaction, the three-fluid magnetohydrodynamics is used. From (17) it follows that the instability growth rate depends on parameters of plasma charged grains.

We consider Saturn's F-ring and laboratory plasma as an application of our theoretical results. Typical parameters of Saturn's F-ring are: $Z_d \sim 10^4$, $n_{0d} \sim 1\,\mathrm{cm}^{-3}$, $n_{0i} \sim 10^4\,\mathrm{cm}^{-3}$, $T_e \sim T_i \sim 1\,\mathrm{eV}$, $B_0 \sim 0.05\,\mathrm{G}$, $m_d \cong 10^{-12}$–10^{-6} g. The instability growth rate and dust cyclotron frequency dependences on a mass of dust particles are shown in Fig. 1 and Fig. 2, respectively.

It is obvious that the maximum value of the instability growth rate $\gamma \cong 10^{-7}\,\mathrm{s}^{-1}$ is reached when a mass of dust particles $m_d \cong 10^{-12}$ g. Cyclotron frequency at the same parameters of dust particles is $\omega_{Bd} \cong 10^{-5}\,\mathrm{s}^{-1}$.

For an estimation of a decrement of the dust KAW we use the expression (12) from [5]:

$$\gamma_L \cong -\sqrt{\frac{\pi}{8}} \left(\frac{c k_\perp}{\omega_{pi}}\right)^2 k_{\mathrm{II}} V_{Ti} \left[1 + \frac{T_i}{\delta T_e}\right]^{-2} \left[1 + \frac{m_e}{m_i} \frac{1}{\delta} \left(\frac{T_i}{T_e}\right)^{3/2}\right].$$

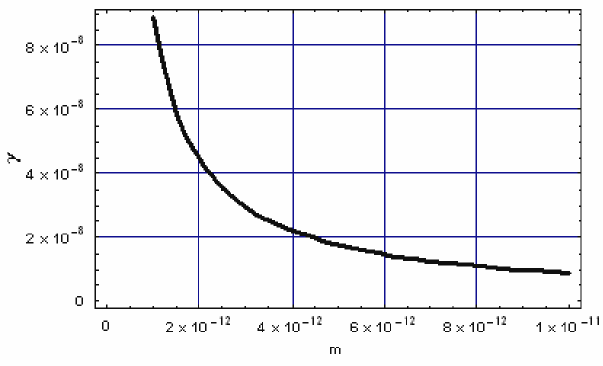

Figure 1. Dependence of the instability growth rate on a mass of dust particles

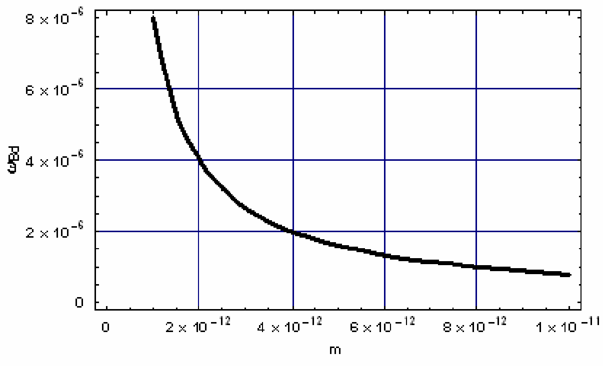

Figure 2. Dependence of the cyclotron dust frequency on a mass of dust particles

Using the same parameters, we obtain decrements which meet the condition $\gamma_L \ll \gamma$. Therefore, Landau damping is enough small, and thus, the considered instability is practically thresholdless.

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