COSMOLOGICAL PARAMETERS FROM OBSERVATIONAL DATA ON THE LARGE SCALE STRUCTURE OF THE UNIVERSE

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The observational data on the large scale structure (LSS) of the Universe are used to determine cosmological parameters within the class of adiabatic inflationary models. We show that a mixed dark matter model with cosmological constant (ΛMDM model) and parameters $\Omega_m = 0.37^{+0.25}_{-0.15}$, $\Omega_\Lambda = 0.69^{+0.15}_{-0.20}$, $\Omega_\nu = 0.03^{+0.07}_{-0.03}$, $N_\nu = 1$, $\Omega_b = 0.037^{+0.033}_{-0.018}$, $n_s = 1.02^{+0.09}_{-0.10}$, $h = 0.71^{+0.22}_{-0.19}$, $b_{cl} = 2.4^{+0.7}_{-0.7}$ (1σ confidence limits) matches observational data on LSS, the nucleosynthesis constraint, direct measurements of the Hubble constant, the high redshift supernova type Ia results and the recent measurements of the location and amplitude of the first acoustic peak in the CMB anisotropy power spectrum. The best model is Λ dominated (65% of the total energy density) and has slightly positive curvature, $\Omega = 1.06$. The clustered matter consists in 8% massive neutrinos, 10% baryons and 82% cold dark matter (CDM). It is shown that the LSS observations together with the Boomerang (+MAXIMA–1) data on the first acoustic peak rule out zero-Λ models at more than 2σ confidence level.

INTRODUCTION

The comparison of recent experimental data on the large scale structure of the Universe with theoretical predictions of inflationary cosmology have shown since quite some time that the simplest cold dark matter (CDM) model is ruled out and we have to allow for a wider set of parameters to fit all observational data on the state and history of our Universe. These include spatial curvature (Ω_k), a cosmological constant (Ω_Λ), the Hubble parameter ($h \equiv H_0/(100\,\mathrm{km\ s^{-1}\ Mpc^{-1}})$), the energy density of baryonic matter (Ω_b), cold dark matter (Ω_{cdm}), the number of species of massive neutrinos (N_ν) and their density (Ω_ν), the amplitude of the power spectra of primordial perturbations in scalar (A_s) and tensor (A_t) modes and the corresponding power-law indices (n_s and n_t), and the optical depth to early reionization (τ). Constraining this multidimensional parameter space, determining the true values of fundamental cosmological parameters, the nature and content of the matter which fills our Universe is an important and exciting problem of cosmology which has now become feasible due to the enormous progress in cosmological observations (short reviews and references see in [18, 7]).

The goal of this paper is to determine cosmological parameters of the sub-class of models without tensor mode and no early reionization on the basis of LSS data related to different scales and different redshifts. We test ΛMDM models with non-zero curvature. Since the sum $\Omega_k + \Omega_\Lambda + \Omega_m = 1$ according to Friedmann's equation, we treat Ω_Λ and Ω_m as free parameters. Furthermore, we use the data on the location and amplitude of the first acoustic peak determined from the most accurate recent measurements of the CMB power spectrum. We also use the SNIa constraint for comparison.

The outline of the paper is as follows: in Sect. 1 we describe the experimental data set which is used here and the method employed to determine cosmological parameters. In Sect. 2 we present and discuss our results. Our conclusions are presented in Sect. 3.

1. THE EXPERIMENTAL DATA SET AND METHOD OF CALCULATION

Our approach is based on the comparison of the observational data on the structure of the Universe over a wide range of scales with theoretical predictions from the power spectrum of small (linear) density fluctuations. The form of the spectrum strongly depends on the cosmological parameters Ω_m, Ω_b, Ω_ν, N_ν, h and n_s. If its amplitude at one given scale is fixed by some observational data then predictions for observations on other scales can be calculated and compared with corresponding observational data. Minimization of the quadratic differences between the theoretical and observational values divided by the observational errors, χ^2, determines the best-fit values of the above mentioned cosmological parameters. For this we use the following observational data set:

1. The location $\tilde{\ell}_p = 197 \pm 6$ and amplitude $\tilde{A}_p = 69 \pm 8\mu$K of the first acoustic peak in the angular power spectrum of the CMB temperature fluctuations deduced from CMB map obtained in Boomerang experiment [3] (these data are sensitive to the amplitude and form of the initial power spectrum in the range $\approx 200\,h^{-1}$Mpc);

2. The power spectrum of density fluctuations of Abell–ACO clusters ($\tilde{P}_{A+ACO}(k_j)$) obtained from their space distribution by [21] (scale range $10 - 100\,h^{-1}$Mpc);

3. The constraint for the amplitude of the fluctuation power spectrum on $\approx 10\,h^{-1}$Mpc scale derived from a recent optical determination of the mass function of nearby galaxy clusters [10] gives $\tilde{\sigma}_8\Omega_m^{\alpha_1} = 0.60 \pm 0.04$ where $\alpha_1 = 0.46 - 0.09\Omega_m$ for flat low-density models and $\alpha_1 = 0.48 - 0.17\Omega_m$ for open models (at the 90% C. L.);

4. The constraint for the amplitude of the fluctuation power spectrum on $\approx 10\,h^{-1}$Mpc scale derived from the observed evolution of the galaxy cluster X-ray temperature distribution function between $z = 0.05$ and $z = 0.32$ derived by [25] $\tilde{\sigma}_8\Omega_m^{\alpha_2} = 0.56 \pm 0.19\Omega_m^{0.1\log\Omega_m+\alpha_2}$, $\alpha_2 = 0.34$ for open models and $\tilde{\sigma}_8\Omega_m^{\alpha_2} = 0.56 \pm 0.19\Omega_m^{0.2\log\Omega_m+\alpha_2}$, $\alpha_2 = 0.47$ for flat models (both with 95% confidence limits);

5. The constraint for the amplitude of the fluctuation power spectrum on $\approx 10\,h^{-1}$Mpc scale derived from the existence of three very massive clusters of galaxies observed so far at $z > 0.5$ [1] $\tilde{\sigma}_8\Omega_m^{\alpha_3} = 0.8 \pm 0.1$, where $\alpha_3 = 0.24$ for open models and $\alpha_3 = 0.29$ for flat models;

6. The constraint on the amplitude of the linear power spectrum of density fluctuations in our vicinity obtained from the study of bulk flows of galaxies in sphere of radius $50\,h^{-1}$Mpc $\tilde{V}_{50} = (375 \pm 85)\,$km/s [14];

7. The constraint for the amplitude of the fluctuation power spectrum on $0.1 - 1\,h^{-1}$Mpc scale and $z = 3$ derived from the Ly-α absorption lines seen in quasar spectra $1.6 < \tilde{\sigma}_F(z = 3) < 2.6$ (95% C. L.) at $k_F \approx 38\Omega_m^{1/2}\,h$/Mpc [11, 22, 12];

8. The constraints for the amplitude and inclination of the initial power spectrum on $\approx 1\,h^{-1}$Mpc scale and $z = 2.5$ scale derived by Croft *et al.* (1998) from the Ly-α forest of quasar absorption lines $\tilde{\Delta}^2_\rho(k_p) \equiv k_p^3 P(k_p)/2\pi^2 = 0.57 \pm 0.26$, $\tilde{n}_p \equiv \dfrac{\Delta\log\ P(k)}{\Delta\log\ k}\,|_{k_p} = -2.25 \pm 0.18$, $k_p = 1.5\Omega_m^{1/2} h$/Mpc (95% C. L.);

9. The data on the direct measurements of the Hubble constant $\tilde{h} = 0.65 \pm 0.10$ which is a compromise between results obtained by two groups [24] and [16];

10. The nucleosynthesis constraint on the baryon density derived from a observational content of inter galactic deuterium $\widetilde{\Omega_b h^2} = 0.019 \pm 0.0024$ (95% C. L.) given by [5];

11. The constraint on the matter and vacuum (cosmological constant) energy density derived from the distance measurements of super novae of type Ia (SNIa) [23, 19, 20] in the form $[\widetilde{\Omega_m - 0.75\Omega_\Lambda}] = -0.25 \pm 0.125$.

One of the main ingredients for the solution for our search problem is a reasonably fast and accurate determination of the linear transfer function for dark matter clustering which depends on the cosmological parameters. We use accurate analytical approximations of the MDM transfer function $T(k;z)$ depending on the parameters Ω_m, Ω_b, Ω_ν, N_ν and h given by [9]. The linear power spectrum of matter density fluctuations $P(k;z) = A_s k^{n_s} T^2(k;z) D_1^2(z)/D_1^2(0)$, where A_s is the normalization constant for scalar perturbations and $D_1(z)$ is the linear growth factor. We normalize the spectra to the 4-year COBE data [2] which determine the amplitude of density perturbation at horizon scale, δ_h [15, 4]. Therefore, each model will match the COBE data by construction. The normalization constant A_s is then given by $A_s = 2\pi^2\delta_h^2(3000\,\text{Mpc}/h)^{3+n_s}$.

The Abell–ACO power spectrum is related to the matter power spectrum at $z = 0$, $P(k;0)$, by the cluster biasing parameter b_{cl}: $P_{A+ACO}(k) = b_{cl}^2 P(k;0)$. We assume scale-independent linear bias as free parameter of which best-fit values will be determine join with other cosmological parameters.

The dependence of the position and amplitude of the first acoustic peak in the CMB power spectrum on cosmological parameters n_s, h, Ω_b, Ω_{cdm} and Ω_Λ can be calculated using the analytical approximation given in [8] which has been extended to models with non-zero curvature ($\Omega_k \equiv 1 - \Omega_m - \Omega_\Lambda \neq 0$) in [7]. Its accuracy in the parameter ranges which we consider is better then 5%.

The theoretical values of the other experimental constraints are calculated as described in [18]. There one can also find tests of the method.

2. RESULTS AND DISCUSSION

The determination of the parameters Ω_m, Ω_Λ, Ω_ν, N_ν, Ω_b, h, n_s and b_{cl} by the Levenberg-Marquardt χ^2 minimization method can be realized in the following way: we vary the set of parameters Ω_m, Ω_Λ, Ω_ν, Ω_b, h, n_s and b_{cl} with fixed N_ν and find the minimum of χ^2. Since N_ν is discrete we repeat this procedure three times for

N_ν=1, 2, and 3. The lowest of the three minima is the minimum of χ^2 for the complete set of free parameters. Hence, we have seven free parameters. The formal number of observational points is 25 but, as it was shown in [18], the 13 points of the cluster power spectrum can be described by just 3 degrees of freedom, so that the maximal number of truly independent measurements is 15. Therefore, the number of degrees of freedom for our search procedure is $N_F = N_{\rm exp} - N_{\rm par} = 8$ if all observational points are used. In order to investigate to what extent the LSS constraints on fundamental parameters match the constraints implied by SNIa [20] we have determined all 8 parameters without (i) and with (ii) the SNIa constraint. In the case without LSS constraints (iii) the number of residual experimental points equals with number of free parameters[1] ($N_F = 0$). The results are presented in Table 1. Note that for all models χ^2_{min} is in the range $N_F - \sqrt{2N_F} \le \chi^2_{\rm min} \le N_F + \sqrt{2N_F}$

Table 1: Cosmological parameters determined from the different set of observational data: (i) – LSS, $\tilde{\ell}_p$, $\tilde{A}_p$, $\tilde{h}$, $\widetilde{\Omega_b h^2}$, (ii) – LSS, $\tilde{\ell}_p$, $\tilde{A}_p$, $\tilde{h}$, $\widetilde{\Omega_b h^2}$, SNIa constraint, (iii) – $\tilde{\ell}_p$, $\tilde{A}_p$, $\tilde{h}$, $\widetilde{\Omega_b h^2}$, SNIa constraint

Data set	χ^2_{min}/N_F	Ω_m	Ω_Λ	Ω_ν	Ω_b	n_s	h	b_{cl}
(i)	5.90/7	$0.37^{+0.25}_{-0.15}$	$0.69^{+0.15}_{-0.20}$	$0.03^{+0.07}_{-0.03}$	$0.037^{+0.033}_{-0.018}$	$1.02^{+0.09}_{-0.10}$	$0.71^{+0.22}_{-0.19}$	$2.4^{+0.7}_{-0.6}$
(ii)	6.02/8	$0.32^{+0.20}_{-0.11}$	$0.75^{+0.10}_{-0.19}$	$0.0^{+0.09}_{-0.0}$	$0.038^{+0.033}_{-0.019}$	$1.00^{+0.13}_{-0.10}$	$0.70^{+0.28}_{-0.18}$	$2.2^{+0.8}_{-0.5}$
(iii)	0/0	0.33±0.07	0.77±0.08	–	0.045±0.014	0.96±0.06	0.65±0.1	–

which is expected for a Gaussian distribution of N_F degrees of freedom. This means that the cosmological paradigm which has been assumed is in agreement with the data. Including the MAXIMA–1 [13] data into the determination of the first acoustic peak does not change the results essentially (see [7]). The errors in the best-fit parameters presented in Table 1 for cases (i) and (ii) are obtained by maximizing the (Gaussian) 68% confidence contours over all other parameters, the errors for the case (iii) are the square roots of the diagonal elements of the covariance matrix.

The model with one sort of massive neutrinos provides the best fit to the data, $\chi^2_{min} = 5.9$. However, there is only a marginal difference in χ^2_{min} for N_ν = 1, 2, 3. With the given accuracy of the data we cannot conclude whether massive neutrinos are present at all, and if yes what number of degrees of freedom is favored. We summarize that the considered observational data on LSS of the Universe can be explained by a ΛMDM inflationary model with a scale invariant spectrum of scalar perturbations and small positive curvature.

Including of the SNIa constraint into the experimental data set decreases Ω_m, increases Ω_Λ slightly and prefers $\Omega_\nu \approx 0$, a ΛCDM model. Excluding the LSS constraints (items 2–8 in Sect. 2) does not significantly change the best-fit values (of those parameters which are determined), which demonstrates nicely the concordance of different experimental data sets and their theoretical interpretation.

In the cases (i) and (ii), the calculated characteristics of the LSS are within the 1σ error bars of the observed values. The predicted age of the Universe agrees well with recent determinations of the age of globular clusters.

Comparing the results obtained without and with the SNIa constraint, we conclude that the values of the fundamental cosmological parameters Ω_m, Ω_Λ and Ω_k determined by the observations of large scale structure match the SNIa test very well. This can be interpreted as independent support of the SNIa result in the framework of the standard cosmological paradigm. However, in order to elucidate how LSS data constraint cosmological parameters, we analyze further only the model obtained without the SNIa constraint.

The best fit values of cosmological parameters determined by LSS characteristics are presented in the 1-st row of Table 1. The best-fit CDM density parameter is $\Omega_{cdm} = 0.30$ and $\Omega_k = -0.06$, slightly positive curvature.

The value of the Hubble constant is close to the result by [16] and [17]. The spectral index coincides with the prediction of the simplest inflationary scenario, it is close to unity. The neutrino matter density $\Omega_\nu = 0.03$ corresponds to a neutrino mass of $m_\nu = 94\Omega_\nu h^2 \approx 1.4$ eV but is compatible with 0 within 1σ. So, ΛCDM model ($\Omega_\nu = 0$) is within the 1σ contour of the best-fit ΛMDM model. The estimated cluster bias parameter $b_{cl} = 2.4$ fixes the amplitude of the Abell–ACO power spectrum.

The errors shown in Table 1 (i) define the range of each parameter within which by adjusting the remaining parameters a value of $\chi^2_{min} \le 11.8$ can be achieved. Of course, the values of the remaining parameters always

[1] Ω_ν and b_{cl} are not determined in this case.

lay within their corresponding 68% likelihoods given in Table 1. Models with vanishing Λ are outside of this marginalized 1σ range of the best-fit model determined by the LSS observational characteristics used here even without the SNIa constraint. Indeed, if we set $\Omega_\Lambda = 0$ as fixed parameter and determine the remaining parameters in the usual way, the minimal value of χ^2 is $\chi^2 \approx 24$ with the following values for the other parameters: $\Omega_m = 1.15$, $\Omega_\nu = 0.22$, $N_\nu = 3$, $\Omega_b = 0.087$, $n_s = 0.95$, $h = 0.47$, $b_{cl} = 3.7$ ($\sigma_8 = 0.60$). This model is outside the 2σ confidence contour of the best-fit model (i). The experimental data which disagree most with $\Lambda = 0$ are the data on the first acoustic peak. If we exclude it from the experimental data set, $\chi^2_{min} \approx 5.8$ for an open model with the following best-fit parameters: $\Omega_m = 0.48$, $\Omega_\nu = 0.12$, $N_\nu = 1$, $\Omega_b = 0.047$, $n_s = 1.3$, $h = 0.64$, $b_{cl} = 2.5$ ($\sigma_8 = 0.82$). This model is within the 1σ confidence contour of the best-fit ΛMDM model obtained without data on the first acoustic peak. The reason for this behavior is clear: the position of the "kink" in the matter power spectrum at large scales demands a "shape parameter" $\Gamma = \Omega_m h^2 \sim 0.25$ which can be achieved either by choosing an open model or allowing for a cosmological constant. The position of the acoustic peak which demands an approximately flat model then closes the first possibility.

Results change only slightly if instead of the Boomerang data we use Boomerang+MAXIMA–1. Hence, we can conclude that the LSS observational characteristics together with the Boomerang (+MAXIMA–1) data on the first acoustic peak already rule out zero-Λ models at more than 95% C. L. and actually demand a cosmological constant in the same bulk part as direct measurements. We consider this a non-trivial consistency check!

Flat Λ models in contrary, are inside the 1σ contour of our best-fit model. Actually, the best fit flat model has $\chi^2_{min} \approx 8.3$ and the best fit parameters $\Omega_m = 0.35 \pm 0.05$, $\Omega_\Lambda = 0.65 \mp 0.05$, $\Omega_\nu = 0.04 \pm 0.02$, $N_\nu = 1$, $\Omega_b = 0.029 \pm 0.005$, $n_s = 1.04 \pm 0.06$, $h = 0.81 \pm 0.06$, $b_{cl} = 2.2 \pm 0.2$ ($\sigma_8 = 0.96$) are close to our previous [18] results with a somewhat different observational data set. It is obvious that flat zero-Λ CDM and MDM models are ruled out by the present experimental data set at even higher confidence limit than by data without the Boomerang and MAXIMA–1 measurements in [18].[2]

3. CONCLUSIONS

The main observational characteristics on LSS together with recent data on the amplitude and location of the first acoustic peak in the CMB power spectrum, and the amplitude of the primordial power spectrum inferred by the COBE DMR four year data prefer a ΛMDM model with the following parameters: $\Omega_m = 0.37^{+0.25}_{-0.15}$, $\Omega_\Lambda = 0.69^{+0.15}_{-0.20}$, $\Omega_\nu = 0.03^{+0.07}_{-0.03}$, $N_\nu = 1$, $\Omega_b = 0.037^{+0.033}_{-0.018}$, $n_s = 1.02^{+0.09}_{-0.10}$, $h = 0.71^{+0.22}_{-0.19}$, $b_{cl} = 2.4^{+0.7}_{-0.7}$ (1σ marginalized ranges).

The central values correspond to a slightly closed ($\Omega_k = -0.06$) ΛMDM model with one sort of 1.4 eV neutrinos. These neutrinos make up about 8% of the clustered matter, baryons are 10% and the rest (82%) is in a cold dark matter component. The energy density of clustered matter corresponds to only 35% of the total energy density of matter plus vacuum which amounts to $\Omega = 1.06$.

The values for the matter density Ω_m and the cosmological constant Ω_Λ for the best-fit model are close to those deduced from the SNIa test. Including this test in the observational data set, results to a somewhat larger value of Ω_Λ (7%) and slightly lowers Ω_m.

The observational characteristics of large scale structure together with the Boomerang (+MAXIMA–1) data on the first acoustic peak rule out zero-Λ models at more than 2σ confidence limit.

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